TIME SYMMETRY AND QUANTUM MEASUREMENTS

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Although the quantum laws are time-reversal invariant, a contradiction appears if two measurements performed by a single observer, and described according to these laws, are performed in two opposite directions of time. This contradiction leads to bringing forth the concept of an observer's private time, and then to building up a temporal parameter common to several observers from their private times. Time asymmetry turns out to be a consequence of the latter construction.

1. Introduction. Although the quantum mechanical law of motion is in most cases as time symmetric as its classical counterpart [1], it is often stated that the measurement process [2] introduces a specific time-asymmetric element in quantum theory [3,4]. This seems to be obvious in the usual interpretation wherein the observation of a quantity described by a linear hermitian operator O_a with eigenstates $|a_i\rangle$ brings about a discontinuous change of the state vector of a system from the initial state $|\psi\rangle$ to an eigenstate $|a_i\rangle$, with predicted probability $|\langle a_i|\psi\rangle|^2$. Several investigators looked for the conditions required to obtain successful "retrodictions" equivalent to predictions when discontinuous transitions to the eigenstates are assumed [5–7]. They found that successful retrodiction is possible for a particular construction of statistical ensembles: the retrodictor has to suppose equal a priori probabilities for the initial states of the systems. The latter assumption is not valid in the general case. But this lack of validity is related to a lack of equivalence between a priori and a posteriori inference [5] which is external to quantum theory.

The situation is different if one considers Everett's interpretation of measurement [8,9]. In this framework one must ascribe a state vector to the observer (or at least to a recording apparatus). Let this vector be $|\eta[...]\rangle$ where the points in the brackets stand for a fixed initial memory configuration. If the initial state of the system is

$$|\psi\rangle = \sum_i c_i|a_i\rangle, \quad \text{with} \quad c_i = \langle a_i|\psi\rangle\,, \tag{1}$$

the initial state of the total system (measured system + observer) is the tensorial product of $|\psi\rangle$ and $|\eta[...]\rangle$:

$$|\psi^{\mathrm{T}}\rangle = \left(\sum_i c_i|a_i\rangle\right)|\eta[...]\rangle\,. \tag{2}$$

After the measurement of O_a, each observer's memory state is correlated with a particular eigenstate $|a_i\rangle$:

$$|\psi^{\mathrm{T}'}\rangle = \sum_i c_i|a_i\rangle\,|\eta[...a_i]\rangle\,. \tag{3}$$

The transition between (2) and (3) obeys the Schrödinger equation. According to Everett [8,9] this transition is to be interpreted as the splitting of the world into several branches, each corresponding to a particular outcome a_i of the measurement. As the splitting takes place in the direction of increasing time, it seems that some time-asymmetric features persist in this interpretation of the measurement process. But the direction of this time asymmetry is in fact arbitrary. Indeed, the continuous change from $|\psi^{\mathrm{T}}\rangle$ to $|\psi^{\mathrm{T}'}\rangle$ according to the Schrödinger equation is invariant under time reversal. $|\psi^{\mathrm{T}'}\rangle$, which is usually thought of as being later than $|\psi^{\mathrm{T}}\rangle$, may be earlier as well. Here again the apparent time asymmetry results from an implicit assumption about the direction of inference.

In this paper we try to clear up the foundations of this assumption. For this purpose, we show in section 3 that although a measurement can, according to quantum laws, be performed in either time direction, there appears a contradiction when a single observer is assumed to perform some measurements in a given time direction and other measurements in the opposite time direction. A careful analysis of this contradiction shows that it is strongly related with the concept of an external time wherein every event can be located, irrespective of the apparatus or observer recording it. This remark leads in section 4 to the introduction of three distinct definitions for the physical entity called time. They are referred to as private time, common time, and external time. This distinction proves to be a good way towards a precise understanding of how the notion of irreversibility and that of time asymmetry may be related. Such a task must be prepared (next section) by finding a formulation of the stages of measurement which does not refer to any direction in external time.

2. Generalizing the concept of preparation of an experiment. In the time-asymmetric conception of measurement, the initial condition (2) is the result of the preparation of a system S and an apparatus A just before the interaction. But the time-reversed conception holds as well: (2) may be the result of the "postparation" of S and A just "after" the interaction, while (3) may be the correlated state just "before" this interaction. The equivalence of both descriptions leads us to replacing the couples (preparation; later observation) and (postparation; earlier observation) by a unique concept. We assume that the statement describing a measurement contains two pieces of information: a "necessary condition" defining the possible outcomes, and a given outcome among the possible ones. In the language of experimental set up, the "necessary condition" refers to the preparation or postparation of apparatus + system, while the outcome is the result observed respectively later or earlier. In quantum theory, the "necessary condition" is denoted by the observable, while the outcome is an eigenvalue of this observable. An observable can be thought of gathering all possible experimental conditions which enable one to measure a given quantity. We may express this by defining a surjective function between the set of "necessary conditions" and the set of observables

$$N_j \xrightarrow{s} O_{k(j)} ,$$

where $k(j)$ is an integer associated to j.

Besides the recording of an outcome, Everett's picture of a measurement should then include the setting down of the corresponding "necessary condition". This can be carried out by introducing a second-order observer who prepares (postpares) the experiment. To describe a measurement in such a way, the first total state vector to write down is

$$|\phi_0^{\mathrm{T}}\rangle = \left(\sum_j |\psi\rangle^{(j)} |\eta[...]\rangle^{(j)}\right) |\mathrm{Z}[...]\rangle , \tag{4}$$

where $|\psi\rangle^{(j)}$ is the state vector of a given system, $|\eta[...]\rangle^{(j)}$ the state vector of the measuring apparatus, and $|\mathrm{Z}[...]\rangle$ the state vector of the second-order observer. Each particular necessary condition is denoted by the superscript (j). Such a condition specifies the apparatus and system under consideration, together with their initial (final) state. Then

$$|\phi_1^{\mathrm{T}}\rangle = \sum_j |\psi\rangle^{(j)} |\eta[...]\rangle^{(j)} |\mathrm{Z}[..N_j]\rangle \tag{5}$$

represents a state in which the second-order observer is aware of (or chooses) the jth necessary condition. The measuring interaction between the system and the apparatus gives

$$|\phi_2^{\mathrm{T}}\rangle = \sum_j \left(\sum_i c_i^{k(j)} |a_i^{k(j)}\rangle\, |\eta[...a_i^{k(j)}]\rangle^{(j)}\right) \times \mathrm{Z}[...N_j]\rangle , \tag{6}$$

where $a_i^{k(j)}$ is the ith eigenvalue of the observable $O_{k(j)}$. At last, the result of an interaction leading the second-order observer to known the outcome registered by the apparatus is

$$|\phi_3^{\mathrm{T}}\rangle = \sum_j \sum_i c_i^{k(j)} |a_i^{k(j)}\rangle\, |\eta[..a_i^{k(j)}]\rangle^{(j)} \times |\mathrm{Z}[...(N_j, a_i^{k(j)})]\rangle . \tag{7}$$

Several other intermediary states might have been considered. For instance, instead of (6) and (7), we could have had

$$|\phi_{1'}^{\mathrm{T}}\rangle = \left(\sum_i \sum_j c_i^{k(j)} |a_i^{k(j)}\rangle \, |\eta[...a_i^{k(j)}]\rangle^{(j)} \right)$$

$$\times \, |Z[...]\rangle ,$$

$$|\phi_{2'}^{\mathrm{T}}\rangle = \sum_i \sum_j c_i^{k(j)} |a_i^{k(j)}\rangle \, |\eta[...a_i^{k(j)}]\rangle^{(j)}$$

$$\times \, |Z[...a_i^{k(j)}]\rangle . \tag{8}$$

The state $|\phi_{2'}^{\mathrm{T}}\rangle$ describes a situation in which the second-order observer is aware of a certain number $a_i^{k(j)}$. But this number cannot have, for this observer, the meaning of an outcome for the measurement of $O_{k(j)}$, since he knows no necessary condition for it. We shall thus assume in the following that a measurement sequence must take the form (4)–(7).

3. Is an observer able to perform experiments in both time directions? Due to the time-reversal invariance of an interaction described by the Schrödinger equation, a measurement sequence of the form (4)–(7) is able to follow the usual direction of increasing time or the opposite one as well. The perfect formal equivalence between the time directions of quantum measurement gives rise to the classical paradox of a time-symmetric theory confronted with a time-asymmetric scientific practice. This would actually be paradoxical if such a formal equivalence between the time directions of measurement allowed any observer to use both. To investigate the latter point, let us suppose that a measurement M_a described by the sequence (4)–(7) follows the subsequent chronology: between t_1 and t_2, the total state is (5). Between t_2 and t_3, the total state is (6), and between t_3 and t_4 the total state is (7). Another measurement M_b is performed at t_4 by the observer whose state vector is $|Z[...(N_j, a_i^{k(j)})]\rangle$. A state $|Z[...(N_j, a_i^{k(j)})]\rangle$ is either the initial state of the observer Z, if M_b takes place forwards in time, or Z's final state if M_b takes place backwards in time.

For one given observer's initial (final) state, this measurement leads to the following sequence:

$$|\phi_0^{*\mathrm{T}}\rangle = \left(\sum_{j'} |\psi^*\rangle^{(j')} |\eta^*[...]\rangle^{(j')} \right) |Z[...(N_j, a_i^{k(j)})]\rangle ,$$

$$|\phi_1^{*\mathrm{T}}\rangle = \sum_{j'} |\psi^*\rangle^{(j')} \, |\eta^*[...]\rangle^{(j')} \, |Z[...(N_j, a_i^{k(j)}); N_{j'}^*]\rangle , \tag{9}$$

$$|\phi_2^{*\mathrm{T}}\rangle = \sum_{j'} \left(\sum_{i'} c_{i'}^{k^*(j')} |b_{i'}^{k^*(j')}\rangle \, |\eta^*[...b_{i'}^{k^*(j')}]\rangle \right)$$

$$\times \, |Z[...(N_j, a_i^{k(j)}); N_{j'}^*]\rangle , \tag{10}$$

$$|\phi_3^{*\mathrm{T}}\rangle = \sum_{j'} \sum_{i'} c_{i'}^{k^*(j')} |b_{i'}^{k^*(j')}\rangle |\eta^*[...b_{i'}^{k^*(j')}]\rangle$$

$$\times \, |Z[...(N_j, a_i^{k(j)}); (N_{j'}^*, b_{i'}^{k^*(j')})]\rangle . \tag{11}$$

The total states (9)–(11) describe the system respectively t_4 and t_5, t_5 and t_6, t_6 and t_7. If the two measurements are performed in the same time directions, i.e. if the times t_1 to t_7 are such that $t_1 > t_2 > t_3 > t_4 > t_5 > t_6 > t_7$, or $t_1 < t_2 < t_3 < t_4 < t_6 < t_7$, no difficulty is met. But let us assume that the first measurement is performed in a given time direction and the other in the opposite one, according to the following scheme: $t_1 < t_2 < t_3 < t_4$, $t_4 > t_5 > t_6 > t_7$ and $t_3 < t_6 < t_4$. In this situation, there appears a contradiction: At any instant t' included within the interval (t_3, t_6), the memory states of the observer are given both by (7) and by (11). What is then the actual set of memory states at t'?

Before answering the latter question, its precise meaning must be investigated. As a starting point, notice that the difficulty which is brought out by this question can be formulated more clearly if the measurement process (4)–(11) is described from two distinct viewpoints.

The first point of view we shall consider is that of an additional observer (let us call him A_0). This observer A_0 is supposed to locate the events and to describe quantum interactions between the system, the apparatus and the observer Z, in a time which is a passive framework, completely independent of the events or observers' memory states being located in it. If two measurements are performed by Z in two opposite directions of this dimension (referred to as "external time" in the following), A_0 cannot decide which set of Z's memory states ((7) or (11)) represents the actual set of possible outcomes he can obtain at t'. From the point of view of the observer A_0, who locates the events in an external time, the previous contradiction thus persists although its formulation is slightly different.

On the other hand, if the point of view of the observer Z is adopted, the situation is completely different. His sequence of memory states is the following, irrespective of the location of each state in the external time:

$$[...] , \quad (12)$$

$$[...N_j] , \quad (13)$$

$$[...(N_j, a_i^{k(j)})] , \quad (14)$$

$$[...(N_j, a_i^{k(j)}); N_{j'}^*] , \quad (15)$$

$$[...(N_j, a_i^{k(j)}); (N_{j'}^*, b_{i'}^{k^*(j')})] . \quad (16)$$

This means that in state (14), Z knows the necessary condition and the outcome of the measurement M_a, whereas in state (16) he knows the necessary conditions and the outcomes of the measurements M_a and M_b. For him, the measurement M_b has either been performed (memory state (16)) or not (memory state (14)). For Z, there is no ambiguity about his own memory state.

It is clear that the above-mentioned contradiction cannot be expressed in terms of the internal memory states of the observer Z who performs the measurements M_a and M_b. This contradiction is only related with the location of the events in an external time.

4. Private time and common time. At this stage, it is necessary to analyse the operational meaning of the concept of "external" time. The investigation starts with a study of what may be called a private time, which refers to a sequence of observer's internal memory states. Then, the question of how to correlate two or more private times to build up a common time will be dealt with.

Private time may be defined as the numerical labeling of the observer's memory states. Let us assume that a label is an interval of real numbers: (t_{p_1}, t_{p_2}). Two minimal conditions must be imposed to this labeling:

(1) Two different memory states correspond to two labels whose intersection may be one number at the most.

(2) If a memory sequence m' is obtained by appending an isolated necessary condition or an outcome to the sequence m, there is an intersection between the label of m and the label of m'.

For instance, if the memory state (14) has the label (t_{p_1}, t_{p_2}) and if (15) has the label (t_{p_3}, t_{p_4}), we must have either $t_{p_2} = t_{p_3}$ or $t_{p_1} = t_{p_4}$ in order to fulfill the two requirements. Moreover, it is easy to show that, according to these requirements, any set of memory states is labeled in such a way that a sequence of increasing information is associated with a sequence of either monotonically increasing or monotonically decreasing real numbers t_p. The direction of private time is thus unique, although it is arbitrary.

A new analysis of the asymmetry between prediction and retrodiction in quantum mechanics can now be undertaken. According to the usual formulation of this asymmetry, quantum probabilistic laws are successful when they apply to the future, while they generally fail when applied to the past. This formulation amounts to relating the previous asymmetry to an external time wherein the words "past" and "future" are defined. Here instead, we will show that the distinction between two directions of quantum inference can be expressed without reference to any external time.

Let us first consider an observer's memory state which contains the result of a measurement of the observable O_a performed on n systems:

$$[...(N_{a_1}, a_{i_1}); (N_{a_2}, a_{i_2}); ...; (N_{a_n}, a_{i_n})] . \quad (17)$$

Next, we write another observer's memory state containing both the informations of (17) and the specification of n' necessary conditions N_{b_k}:

$$[...(N_{a_1}, a_{i_1}); ...; (N_{a_n}, a_{i_n}); N_{b_1}; ...; N_{b_{n'}}] . \quad (18)$$

These necessary conditions N_{b_k} describe the initial (final) state of an apparatus allowing to measure an observable O_b which does not commute with O_a, together with the initial (final) state of the n' systems on which the measurement of O_b is to be performed. We can suppose (for instance) that these n' systems are such that the measurement of O_a is associated to the particular outcome a_i. Finally, we consider the memory state containing the informations of (18) and the outcome of the n' measurements of O_b:

$$[...(N_{a_1}, a_{i_1}); ...; (N_{a_n}, a_{i_n});$$

$$(N_{b_1}, b_{i'_1}); ...; (N_{b_{n'}}, b_{i'_{n'}})] . \quad (19)$$

The quantum laws can be compared directly to the results contained in the memory sequence (19). The formula $P(i \to i') = |\langle a_i | b_{i'} \rangle|^2$ gives the expectation value of the frequency of the outcome $b_{i'}$ associated to a measurement of O_b on a set of systems whose initial (final) state, specified in the necessary conditions N_{b_k}, is $|a_i\rangle$. This type of quantum probabilistic description is generally successful. On the other hand, the reverse formula

$$P(i'' \leftarrow i') = |\langle b_{i'}|a_{i''}\rangle|^2$$

fails to give the expectation value of the frequency of an initial (final) state $a_{i''}$, specified in the necessary conditions N_{b_m} which are associated with a given outcome $b_{i'}$. Such a failure is quite obvious in the instance we considered above, since all the necessary conditions N_{b_m} specify that their n' systems are in the unique initial (final) state $|a_i\rangle$. The quantum laws are thus successful when they operate from the necessary condition to the outcome, i.e. in the direction of increasing information, while they generally fail in the reverse order. But this order bears no relationship with any external time order. Moreover, as shown previously, it is not necessarily related to an increase of the private time parameter t_p. To denote the asymmetry of quantum inference, it is thus important to insist on information increase which is the fundamental concept, rather than on time ordering.

When the usual couple prediction/retrodiction expresses this asymmetry, it is implicitly assumed that information increases in the direction of increasing time (without any precision about the private or external character of this time). If, instead, the information increases with decreasing (private) time, another couple of words must be used. We suggest postdiction/antediction. But in so far as information increase is dealt with, the terms prediction and postdiction are equivalent, as well as the terms retrodiction and antediction. To emphasize the absence of any relation between the asymmetry of quantum inference and time increase, we shall replace the traditional distinction prediction/ retrodiction by the distinction PP-diction/AR-diction which does not refer to time.

Comment. The above-mentioned concept of retrodiction (antediction) only refers to what can be said about the content of the memory sequences. It does not bear on the event which "really" happened (will happen) in the past (future), and was (will be) presumably recorded in the memory sequence. The giving up of the concept of direct retrodiction (antediction) in the framework of Everett's interpretation of quantum theory was demonstrated to be a possible way to account for the EPR correlations without any violation of the locality principle [10].

We have seen that the direction of quantum inference can be arbitrarily associated to increasing or decreasing private time. The question now is to know whether the same arbitrariness is still available when the observers locate the events in the same time. In other words, are two observers able to perform their experiments in two opposite directions of the same time?

A frequent way to answer this question consists in noticing that two observers who gain information in two opposite directions of (external) time cannot communicate with each other [11]. This simple claim has recently been dismissed, and a procedure allowing two observers, who "have not the same direction of time", to exchange information, has been brought out [12]. The very meaning of the sentence "two observers go in the same (or in opposite) direction(s) of time" however seems not to have been seriously investigated in ref. [12]. It is this meaning to which we now try to give a precise formulation, by comparing the observers' time-directions of measurement. The increase or decrease of the t_p-parameters involved in the private time of two observers, with increasing information in their memory sequences, cannot be taken as a satisfactory basis for such a comparison. Indeed, there is no operational proof making certain that the t_p-parameters used by the observers to label their memory states refer to a common ordering of external events. The observers can however coordinate their private times by performing measurements on quantum objects which we shall call "elementary clocks". When the observer 1 measures any physical quantity on his elementary clock, described by a pure state, he brings about a transition of the state towards a mixture. Such a transition, which is associated with either increasing or decreasing values of the parameter t_p, yields an increase of the entropy S_c of the clock ($S_c = \mathrm{Tr}(\rho_c \log \rho_c)$ where ρ_c is the density matrix of the clock.). Now, as we already know, two measurements performed by a given observer are associated with an unique sign of variation of the parameter t_p. An information exchange between two observers can be thought of as a mutual measurement. In particular, the information increase of the observer 1 about the observer 2, and the entropy increase of 1's clock, are necessarily associated to the same variation of t_p. The same can be said of the observer 2 performing measurements on his own elementary clock, and on observer 1. Whatever t_p-parameter they use privately, two interacting observers are thus bound to note that their clocks experience an entropy increase in the direction of mutual measurement.

This reasoning shows that, although the assertion according to which two observers who have not the same direction of (external) time cannot communicate is false, it can be said that the very act of communication (with its meaning of mutual quantum measurement) defines a link between the information increase of an observer about the other one, and the entropy increase of any quantum object. This link can be used by the observers to establish a common time direction, by relating it to the entropy increase of their elementary clocks.

In this paper, we have first noticed that in so far as time is considered as a parameter independent of the observer, the latter can perform a quantum measurement in whatever direction of this time. But if a given observer is supposed to perform some measurements in a definite direction of this time, whereas he performs other measurements in the opposite direction, there appears a contradiction. This contradiction has brought out the necessity to introduce a concept of private time. Then, it was demonstrated that the very definition of a time common to two or more observers prevents any observer from performing measurements in opposite directions of the latter time. The essential difference between a common time and the usual concept of external time independent of any observer is thus that asymmetry is built in the first one, whereas it must be superimposed to the second one. But the direction of this superimposed asymmetry is arbitrary. As Schrödinger pointed it out, all the statements about irreversibility may even be formulated in such a fashion that they are (external) time-reversal invariant [13]. This is no more true when common time is considered.

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References

[1] A. Messiah, Mécanique quantique, Vol. 2 (Dunod, Paris, 1960).
[2] B. d'Espagnat, Conceptual foundations of quantum mechanics (Benjamin, New York, 1976).
[3] A. Daneri, A. Loinger and G.M. Prosperi, Nucl. Phys. 33 (1962) 297.
[4] L. Rosenfeld, Prog. Theor. Phys. Suppl. (1965) 222.
[5] S. Watanabe, Rev. Mod. Phys. 27 (1955) 26.
[6] Y. Aharonov, P.G. Bergmann and J.L. Lebowitz, Phys. Rev. B 134 (1964) 410.
[7] P.C.W. Davies, The physics of time asymmetry (University of California Press, Berkeley, 1977).
[8] H. Everett, Rev. Mod. Phys. 29 (1957) 454.
[9] H. Everett, in: The many-worlds interpretation of quantum mechanics, eds. B.S. De Witt and N. Graham (Princeton Univ. Press, Princeton, 1974).
[10] M. Bitbol, Phys. Lett. A 96 (1983) 66.
[11] J.R. Lucas, A treatise on time and space, (Methuen, London, 1973).
[12] M. McBeath, Synthese 56 (1983) 27.
[13] E. Schrödinger, Proc. R. Ir. Acad. 53 (1950) 189, in: The enigma of time, ed. P.T. Landsberg (Hilger, London, 1984).