

The constitution of objects in classical physics and in quantum physics

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1. Introduction

The present article is concerned with constitution of objects in physics. It leads from Kant's transcendental arguments in the *Critique of Pure Reason* to the concept of objects in classical physics and in quantum physics. The investigations are based on the surprising observation, that the method of constituting objects in Kant's critical philosophy can be applied almost literally to classical physics, and with some small but essential restrictions also to quantum physics. Hence, one could get the impression that there is a strong continuity in the history of the foundations of physics. This is, however, not the case. Neither in the highly developed formulation of classical mechanics in the 19th century, nor in the original version of quantum mechanics, which was formulated in 1925-32, the method of constituting objects was applied. Instead, objects were considered in both cases as elementary entities, as mass points, massive bodies, or particles, and these objects were inserted *ad hoc* into the already completely formulated theories. Only during the last decades it became obvious how objects can be introduced systematically into the two fields of physics mentioned and that this incorporation is in fact an adoption and realisation of Kant's original transcendental way of reasoning.

2. The Cognition of Objects in Kant's Philosophy.

2.1 Historical preliminaries

In the *Critique of pure reason*¹ Kant formulated his transcendental philosophy in contrast to two alternative positions, the metaphysics of Leibniz and Wolff, and the empiricism of Hume and Locke. In particular, for the problem of constituting objects the sceptical philosophy of Hume is most interesting as an opposite project. As to the question whether in addition to our direct perceptions there are objects in the external reality and what we know about these entities, Hume presented his opinion at different places. The importance of this problem becomes obvious, if we realise that the directly observable and usually time dependent qualities or predicates are per se not a

criterion of an object, since the various properties are in general different from each other. Accordingly, Hume writes that this difference

*“obliges the imagination to feign an unknown something, or original substance and matter, as a principle of union and cohesion among these qualities, and as what may give the compound object a title to be call’d one thing, [...]”*²

Indeed, the qualities that we observe are first of all completely independent of another and independent of a carrier whose properties they probably are:

*“Every quality being a distinct thing from another, may be conceiv’d to exist apart, and may exist apart, not only from every other quality, but from that unintelligible chimera of substance”.*³

Hence, we must ask, whether the search for an object is nothing but hunting a chimera. - Kant has taken up this question and answered it within the framework of his transcendental philosophy. However, this question must also be answered within the framework of any field of science that claims to be a realistic description of nature.

2.2 *The constitution of objects in Kant’s Philosophy*

Also Kant’s way of reasoning begins with the argument that our perceptions lead per se merely to the cognition of qualities but not to objects whose properties correspond to the observed qualities, and which are the time-independent carriers of time-dependent properties. In this point, Kant agrees with Hume. However, Kant doubts that objects are merely the products of our imagination and he gives two reasons. First, it is not always possible to relate several observed qualities to an object as their referent. This is only the case, if the observed qualities fulfil some necessary conditions. With respect to this argument, Kant is more cautious than Hume. However, if the necessary preconditions mentioned are fulfilled, then, according to Kant, the object that persists in time in contrast to the time-dependent properties is an element of the objective and external reality – and not a “chimera”. Obviously, in this point Kant exceeds Hume’s empiricist position.

The constitution of “objects of experience” from our perceptions and observations starts with the requirement of *objectivity*. The observed qualities should not refer to the perceiving subject but to the objective, external reality, which is clearly distinguished from the subject. In order to apply this realistic interpretation to our observations, some necessary conditions must be fulfilled. It should be possible to order and to interpret the observed qualities according to some conceptual

¹ Kant, (1998), CpR.

² Hume, 1978 (1748), p.221

³ Hume, 1978 (1748), p.222

prescriptions, the categories of substance and causality. It should be possible to consistently relate the time-dependent predicates to a substance as the time-independent carrier of the predicates in question. And in addition, it should be possible to interpreting the temporal changes of the predicates as causal alterations of the properties of the object.

Kant does not state that an interpretation of this kind can always be applied to our perceptions and sensations. But he claims that if this is impossible, then there is no cognition at all.⁴ However, if our observations refer to an element of the external reality and not to the observing subject, then the observations in space and time must have been ordered and interpreted according to the categories of substance and causality. In this way, “objects of experience” are constituted and the categories mentioned are necessary preconditions of these objects that – for this reason – fulfil the *a priori* laws of substance and causality. In other words, there is a time-independent carrier of time-dependent properties, whose temporal alterations obey some causal regularity. Hence, an object of experience is an element of the external reality that is clearly distinguished from the observer. The observable and changeable qualities can be related to this object, which itself is determined by a few unchangeable, permanent features.⁵

The categories of substance and causality belong to the necessary preconditions of objects of experience. However, the constitution of objects by means of these categories determines in general merely the kind of objects that are characterised by some permanent properties, but not individuals. For the determination of individual objects, we must extend the *formal* preconditions of experience, in particular the categories mentioned, by *material* preconditions of experience. The material preconditions of experience correspond to the material possibilities to perform observations of predicates and they extend the possibilities for constituting objects. In this context, Kant’s “principle of complete determination”, which applies to “things”, becomes relevant:

*“Every **thing**, however, as to its possibility, further stands under the principle of **thoroughgoing determination**; according to which, among **all possible predicates of things**, insofar as they are compared with their opposites, one must apply to it.”⁶*

This principle does not follow from the preconditions of experience. However, if it can be fulfilled, it allows for further determination of objects. In particular, the position property pertains to a “thing” at any time. Objects that possess the position-property at any time will be called here “continuously localizable”.

⁴ Kant, (1998), CpR., pp. 227-228.

⁵ Here we could think of the mass, the form, etc. Cf. also Kant, (1998), CpR., p. 379.

⁶ Kant, (1998), CpR., p. 553.

Even if we presuppose “continuous localizability”, the determination of individual objects by their positions is not possible in general, since two objects that are equal with respect to all other predicates, could still be at the same place. Hence, for the determination of individual objects we must assume in addition, that objects possess the contingent property of *impenetrability*. Kant mentioned the possibility of individuation by means of the position property only casually and without taking account of the impenetrability, when he put forward his critique of Leibniz’ “principium identitatis indiscernibilium”:

“[...] *then the issue is not the comparison of concepts, but rather, however identical everything may be in regard to that, the difference of places of these appearances at the same time is still an adequate ground for the **numerical difference** of the object (of the senses) itself.*”⁷

Kant’s considerations show, which necessary and which contingent preconditions must be fulfilled in order to consistently relate the observed qualities to an object as their carrier. This method of constituting objects at all, and in particular individuals, must be concretised in the various fields of natural sciences. We should not expect, that in these fields, objects can be determined in a less complicated way.

3. Objects in Classical Physics

3.1 *Historical Preliminaries*

Kant’s reaction to the empiricism of David Hume had shown, in which way cognition of “objects of experience” can be achieved and that in spite of the sceptical arguments mentioned. Hence, one could guess that in classical physics, and in particular in classical mechanics, the Kantian way of reasoning would have been adopted in order to guarantee the objectivity of cognition in physics and to characterise the concept of a mechanical object in the transcendental way. This was, however, not the case.

In 1787, when the *Critique of Pure Reason* appeared, the most important field of physics was Newton’s mechanics, first published in 1687, and further elaborated by d’Alambert (1758), Lagrange (1788), etc. From a philosophical point of view, this theory was, however, still exposed to the objections against a theory in the sense of empiricism. In particular, this means that objects are not constituted within the theory on the basis of observable qualities, but inserted into the theory of predicates as primitive entities.

⁷ Kant, (1998), CpR., p. 368. Cf. also pp. 372-373.

Since Hume's scepticism was one of the starting points of Kant's critical philosophy, a reformulation of classical mechanics on the basis of Kant's transcendental way of reasoning would have suggested itself. However, this idea is confronted with serious difficulties. Within the framework of Lagrange's formulation of classical mechanics, the constitution of objects would have been extremely difficult, if not impossible. Only in the new and more advanced formulation of classical mechanics by Hamilton (c 1835), there was some chance for applying the idea of constituting objects in the new "canonical formalism" of mechanics.⁸ But even in this "phase-space" formulation of classical mechanics, not all the tools necessary for constituting objects were already available. We mention here, in particular, the theory of "Continuous Groups of Transformations", which was developed by Sophus Lie not before the end of the 19th century.

Except from these technical questions, we should mention that the philosophical situation had changed very much in the last decades of the 19th century. Neither Hume's arguments against objects nor Kant's reaction to this position were seriously discussed in the philosophy of physics. Instead, empiricism and positivism were considered as an adequate philosophical basis of physics. Kant's critical arguments, and in particular the idea of constituting objects, were almost ignored at that time. For these reasons, it is no surprise that classical mechanics was not reformulated in the sense of Kant's critical philosophy but considered as a field of science that is based philosophically on empiricism or on positivism.

The open systematic problem, how we can get objective knowledge of things or objects in physics, was treated not before 1963 and that first as a mathematical problem of quantum mechanics, in spite of the fact that in quantum mechanics the formal problems are even more difficult than in classical mechanics.⁹ Here, we will not follow the historical development of physics but investigate first (in 3.3) how objects can be constituted in classical mechanics. Kant's arguments can be applied here almost literally. In a second step (in 4.3), we treat the same problem in quantum mechanics and find that we are confronted here with new difficulties unknown in Kant's philosophy and in classical mechanics.

3.2 Objectivity and Invariance

⁸ It should be added that W. R. Hamilton (1805-1865) was quite familiar with Kant's Critique of Pure Reason.

⁹ Cf.: Makey, G. (1963); Sudarshan, E.C.G./Mukunda, N., (1974); Piron, C. (1976).

Classical mechanics describes the properties of classical objects and in particular the time dependence of properties. The mathematical framework of mechanics – in the Hamiltonian formulation – is the space of possible states of an object system, the “phase-space”. Observables are then given by convenient functions on this phase space. In this Hamiltonian formulation, classical mechanics is still exposed to the critique of the empiricism. Since the theory is concerned merely with observables and their time dependence the concept of an object as the carrier of the observable properties is almost void. Indeed, Hume’s critique applies to this theory almost literally since an object is an “unknown something”, a product of our “imagination”, but not an element of the theory.

According to Kant, we start with the requirement of objectivity of our cognition. Similarly, in physics, and in particular in classical mechanics, our goal is the cognition of the external reality and not of the observing subject. Accordingly, observations or measuring results should refer to the external reality, and not to the observer and his impressions. Hence, the cognition of the external reality must be independent in some sense of the preconditions of the observer. The subjective, observer dependent component of an observation or a measurement result is given by the space-time coordinates of the observer. Hence, the requirement of objectivity means that the laws of the external reality must fulfil some *invariance* properties.¹⁰ If an observer changes his space-time coordinates, then the observations should be changed such that they refer to the same but equivalently changed object. The same changes can also be obtained, if the object is subject of an *active* transformation that corresponds to a *passive* transformation of the coordinate system. Weyl illustrates this symmetry of active and passive transformations by a simple geometrical example.¹¹ For geometrical objects like triangles in the Euclidean plane, we have always symmetry between active and passive transformations. However, for physical objects this symmetry is a necessary precondition of their objectivity.

Within the context of classical mechanics, these relations can be made more explicit. The fundamental laws of classical mechanics are invariant with respect to the transformations of the 10-parameter Galileo group G . For a given inertial frame of reference, these transformations consist of 3 translations in space, 3 rotations in space, 3 changes of the constant velocity of the inertial system and 1 translation in time. If the observer is "moved" in accordance with a Galileo transformation, then the observations, which refer to the external object, will transform "covariant" with respect to this transformation. Since also the observers, represented by measurement instru-

¹⁰ This point was emphasised first by Weyl (1927)

¹¹ Weyl, (1927), pp.88-89

ments are physical objects, they will be subject to the same invariance laws. This implies a symmetry between active and passive transformations: The transformation of the measurement results does not depend on whether the observer is moved according to a Galileo transformation or whether the object is moved according to the inverse transformation.

3.3 Covariance and Observables

The symmetry between active and passive transformations allows for clarification of the concept of an "observable". Intuitively, an observable is a measurable quantity or a property of an object system S , that belongs to the external reality and that is clearly distinguished from the observer and the apparatus, respectively. "Properties" (or predicates) may pertain to the object or not, and hence they correspond to value definite yes-no propositions P_i or to the most simple observables with only two values 0 and 1, say. The set $\{P_i\}$ of elementary propositions can be extended by introducing the logical operations \wedge , \vee , \neg , and the relation \leq . In this way, we arrive at the full propositional system of classical mechanics, which is given by the complete, atomic and Boolean lattice L_C of classical logic.

One can then define an "observable" as a relation between numbers on the reading scale of the apparatus and properties of the object system. Hence, an observable may be considered as a mapping Φ from the Borel sets $B(\mathfrak{R})$ of the real numbers \mathfrak{R} (of the reading scale) to the Boolean lattice L_C of propositions. An observable is connected with the group G of Galileo transformations in a two-fold way. On the one hand, the properties of the system S are changed by an *active* transformation, when the transformation group acts on the system and its propositional lattice. On the other hand, the coordinate system of the observer is changed by a *passive* transformation, when the transformation group acts on the measurement device M , i.e. on the Borel sets of the reading scale.

Within this conceptual framework, the symmetry between active and passive transformations leads to the following important *covariance postulate* (C), which must be fulfilled by an observable Φ that can be interpreted as a property of a really existing object: The properties $\Phi[B(\mathfrak{R})]$ of the object S that are *actively* transformed by a representation $S(G)$ of the *Galileo* group must coincide with the properties $\Phi[B(\mathfrak{R})]$ that one obtains from Borel sets $B(\mathfrak{R})$ (of the reading scale of the apparatus M) that are *passively* transformed by a representation $M(G)$ of the *Galileo* group. This means that the diagram in Figure 1 must „commute“. The covari-

ance postulate (C) determines those functions Φ , which may be considered as „observables“ and it shows, how these observables are transformed under a special transformation.¹²

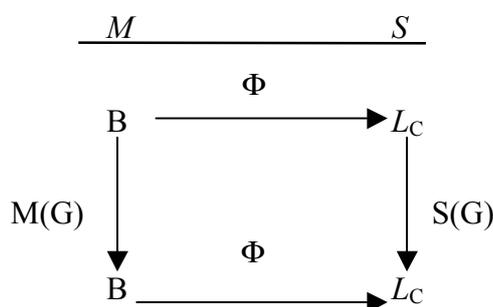


Fig. 1. Covariance diagram of classical mechanics

On the basis of the covariance postulate (C) and the Galileo group, one can now define the fundamental observables p (momentum), q (position) and the observable t (time). In this way, the basic quantities (p , q , t) of the state space can be shown to be "observables" in the sense of the covariance postulate. If an object of classical mechanics is understood as a carrier of properties, then it is obviously sufficient, to require that it is a carrier of the fundamental observables p , q , and t .

3.4 Classical objects

A *classical* object is a carrier of the properties $P \in L_C$, not only in one contingent situation K given by the observers system of coordinates, but also in all other situations K' that evolve from K by Galileo transformations, - where the properties are transformed under these transformations according to the covariance postulate. Mathematically, these objects are representations of the Galileo group. One can further specify this concept by considering different classes. Elementary systems, say, are given by irreducible representations of the Galileo group. For elementary systems that correspond to mass points without geometrical structure, there are no *true* but only *projective* representations of the group G . These representations are characterised by one continuous parameter m , which can be interpreted as the "mass" of the object. The next, slightly more general system is a rotating system, with three additional degrees of freedom which correspond to the components of the internal angular momentum.¹³

3.5 Individual systems

¹² Considerations of this kind can be found in Makey (1963) and in Piron (1976). The connection with Kant's philosophy is established in Mittelstaedt (1994) and (1995).

¹³ For more details cf. Sudarshan et al. (1974), pp.389 ff.

The representations of the Galileo group characterise classes of objects with the same permanent properties. In order to denote an *individual* system one has to find additional properties that distinguish the system S in question from all the other systems of the same class. Firstly, one has to make clear, whether the triple (p, q, t) is a unique denotation of S , i.e. whether there is only one system with these properties. Secondly, if *uniqueness* is given, one has to find out in which way the system S defined at time t can be reidentified at some later time t' . In order to guarantee *uniqueness* of S one needs an additional dynamical principle that excludes that two systems are at the same time t at the same phase point (p, q) . Clearly, this postulate is fulfilled if *impenetrability* in position space is given. In order to guarantee also the *reidentifiability* of the system S uniquely defined at time t , at a later time value t' , one needs a convenient law which connects the point $(p, q)_t$ in phase space (at time t) with the phase point $(p, q)_{t'}$ (at any other time t'). In classical mechanics, a dynamical law of this kind is given by a Hamiltonian $H(p, q)$ and the canonical equations. This means that an individual system S can be reidentified at any other time value t' by the (p, q) -values on its dynamical trajectory in phase space. Both requirements for individual objects, the *uniqueness* and the *reidentifiability* are usually guaranteed in classical mechanics. Hence, we can name an individual system S permanently by an arbitrary point (p_t, q_t) on its trajectory.

4. Objects in Quantum Mechanics

4.1 General Remarks

In the “Copenhagen interpretation” of quantum mechanics Niels Bohr made use of an empiricist view and considered only measurement results but without assuming that the observed predicates can be attributed to an object as its properties. Bohr used this “Copenhagen interpretation” not for philosophical reasons, but since the assumption of objects, as carriers of properties is – in general - incompatible with quantum mechanics. The reason why the incorporation of objects is impossible is, that quantum systems are not subject to the “principle of complete determination”. Quantum theory of measurement does not allow for determining jointly all possible properties of a given system. In any contingent situation, described by a state Ψ , only a subset P_Ψ of properties P^i can be measured jointly on the system S . The properties $P^i \in P_\Psi$ are mutually *commensurable*, i.e. they can be measured in arbitrary sequence without thereby changing the results of the measurements. The measured properties can be related to the object system just as in classical mechanics. Hence, we refer to these properties as the “objective” properties of the system in the state Ψ . However, for any state Ψ there are also non-objective properties $P^i \notin P_\Psi$ whose measurement changes the state Ψ of S .

In quantum physics as well as in classical physics for the constitution of objects we begin with the requirement of objectivity. The observed predicates should refer to an object as its properties. Again, this requirement leads to the necessary preconditions of any objective experience, the categories of *substance* and *causality*. However, in the present case the *material* preconditions of classical experience are not fulfilled, since the systems are not "completely determined". From these arguments it follows that the causality law in quantum mechanics holds only for the set P_ψ of objective properties of the state $\Psi(t)$ at a time value t . The time development of this state is governed by the Schrödinger equation and the state $\Psi(t)$ determines the state $\Psi(t')$ at any later time t' . However, since the state Ψ corresponds only to the restricted set P_ψ of objective properties, at different time values we have different sets of objective properties. Hence, it will in general not be possible to establish a causal connection between a property $P^a(t)$ at time t and the same property $P^a(t')$ at a later time t' . Consequently, there is only a very limited *quantum causality* law between the objective properties P_ψ and $P_{\psi'}$ at different time values.¹⁴ Also, Kant's law of the conservation of substance cannot be valid for "all appearances" and must be restricted to the objective properties $P_{\psi(t)}$.

4.2 Objectivity and Invariance

In principle, the same way of reasoning which allows for the constitution of objects in classical mechanics can be applied to quantum mechanics. As in classical mechanics, also in quantum mechanics we are interested in the cognition of the external reality and not in the observing subject. This leads again to the requirement of *objectivity* which means that the fundamental laws of physics are subject to a group of symmetry transformations. Different observers, which are connected by transformations of the invariance group, will then describe the same object of the external reality. The invariance group is again the Galileo group G . The observer corresponds to a classical apparatus, which is associated with a space-time coordinate system. For this reason, the meaning of a passive Galileo transformation is quite similar to the classical case. Different observers are connected by transformations of the Galileo group and the measuring results will then transform "covariant" with respect to these transformations.

As in classical mechanics, observables will be characterised by their covariance with respect to the Galileo group. A Galileo covariant observable can be defined as self-adjoint operator or a projection valued measure Φ . Observables of this kind allow for measurements of proper-

¹⁴ Cf. Mittelstaedt (1994)

ties, they are, however, subject to the well-known complementarity restrictions. The properties of a quantum system S at a some time that correspond to yes-no propositions P_i are given by subspaces of the Hilbert space of the system, or by projection operators. If the set $\{P_i\}$ of propositions is extended by the quantum logical operations \wedge , \vee , \neg and the implication relation \leq , then one arrives at the complete, atomic and orthomodular lattice L_Q of *Quantum Logic*.¹⁵

A quantum mechanical observable Φ is a relation between pointer values Z on the reading scale of the apparatus M and properties of the object system S . Accordingly, an observable is a mapping $\Phi: B(\mathfrak{R}) \rightarrow L_Q$ from the Borel sets $B(\mathfrak{R})$ on the real line \mathfrak{R} to the propositional lattice L_Q , i.e. a projection valued measure. An observable is again connected with the invariance group G in a twofold way. The transformation group acts *actively* on the system changing its properties and it acts *passively* on the measuring outcomes corresponding to Borel sets $B(\mathfrak{R})$. The principle of covariance implies again the equivalence of *active* and *passive* transformations^{16, 17}. Hence, the image $\Phi(Z')$ of a transformed pointer value Z' agrees with the transformed image $\Phi(Z)'$ of the pointer value Z , i.e. the diagram in Fig. 2 „commutes“.

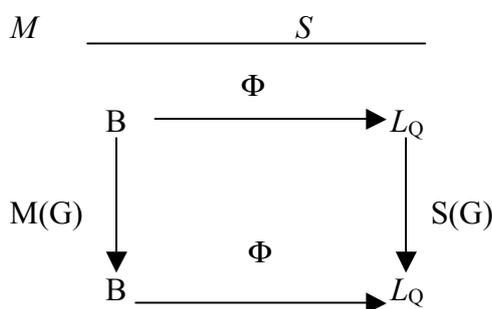


Fig. 2. Covariance diagram of quantum mechanics

The difference between the covariance postulates of classical and quantum physics consists in the different propositional systems L_C and L_Q . The general concept of an observable can again be specified by the fundamental observables q (position), p (momentum) and t (time).

4.3 Quantum objects

A quantum objects is a carrier of the properties $P \in L_Q$, not only in one contingent situation K , given by the observers space time coordinates, but also in all other situations K' that can be

¹⁵ Cf. Mittelstaedt (1995)

¹⁶ C. Piron (1976) p. 93 ff

¹⁷ Mittelstaedt (1995)

obtained from K by Galileo-transformations – where the properties $P \in L_Q$, are transformed covariant under these transformations. In spite of the similarities in the method of constitution, between classical objects and quantum objects there are striking differences that come from the different lattices L_C and L_Q . The propositional system L_C is a complete atomic Boolean lattice. Hence, the object S possesses any property $P \in L_C$ either in the affirmative or in the negative sense, i.e. the object S is *completely determined*. In contrast to this well known situation, a quantum object S possesses at a certain time t simultaneously only a limited class of commensurable properties given by elements of a Boolean sublattice of L_Q . Hence, a quantum system is only carrier of a class of mutually commensurable properties. One can again specify this concept by considering different classes. Elementary quantum systems are given by irreducible unitary representations of the Galileo-group. For elementary objects, there are only projective representations that are characterised by one continuous parameter m which can be interpreted as the mass of the quantum object and which characterises a certain class of objects.

4.4 Individual Quantum Systems

The characterisation of individual objects in quantum mechanics provides problems, that are different from those discussed by Leibniz, Locke, and Kant. The reasons are that – in contrast to Leibniz, – the essential properties are not sufficient for the characterisation of an object and that – in contrast to Locke and Kant – the totality of all accidental properties that were needed for the individualisation is not simultaneously available. Since only *some* classical properties pertain simultaneously to a quantum system, the determination of quantum systems by their accidental properties is never complete. Hence, the characterisation of individual quantum systems by their *permanent* properties fails since the permanent properties define classes of objects, and the characterisation of individual systems by their *accidental* properties cannot be applied, since the accidental properties are not simultaneously available.

In classical physics, the determination of individuals requires *uniqueness* and *reidentifiability*. *Uniqueness* can be achieved only by a property that is subject to some ”generalised impenetrability” which means, that two numerically different objects cannot possess the same value of that property. *Reidentifiability* means that a measurement of the “individuation property” must be repeatable, since otherwise an object, which was determined by this property at a time t could not be re-identified at a later time t' . Since *impenetrability* is known to hold for the position observable, in classical physics the position property is used for the determination of individuals. Since *re-*

peatability does not provide serious problems, for the permanent characterisation of objects trajectories can be used.

In quantum mechanics, the position observable fulfils the impenetrability requirement too and it fulfils the covariance condition with respect to the Euclidean group. However, in the quantum theory of measurement it is well known that repeatability implies discreteness of the measured observable.¹⁸ Since the position observable is continuous, it cannot be measured repeatable and hence it is not possible to re-identify an object by measurement of its position. There are, of course, procedures to discretize a continuous observable. However, a discretization of the position observable would destroy the covariance with respect to the Euclidean group. It is obvious, that the Euclidean covariance must be fulfilled if the position observable shall pertain to the system as an objective property. - Hence, individual objects cannot be determined in quantum physics.

5. Concluding remarks

The transcendental way of reasoning, which was applied by Kant, shows in which way constitution of objects can be achieved. Kant formulated the necessary preconditions that must be fulfilled by the received data if they represent the cognition of an object. We applied these arguments to two different domains.

1. In the domain of classical physics the transcendental strategy can exactly be applied and it leads to the constitution of classical individual objects, provided these objects are impenetrable or characterised by another uniquely determined property.
2. In the domain of quantum physics, the same strategy leads to the constitution of quantum objects, but only to classes of the same kind and not to individual objects. In quantum physics, the constitution of individual objects in the strict sense is not possible.

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¹⁸ Busch et al (1996)