AN ANALYSIS OF THE EINSTEIN–PODOLSKY–ROSEN CORRELATIONS IN TERMS OF EVENTS

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It is shown that Bell's inequalities can be derived from two general assumptions bearing on events, namely the possibility of their retrodiction and their locality. The meaning of a possible violation of the retrodiction principle is interpreted by making use of the many-worlds formulation of quantum mechanics.

The quantum-mechanical predictions about the state of two distant systems which had interacted at some time in the past have been much discussed since the original paper of Einstein, Podolsky and Rosen [1]. Consider Bohm's [2] thought-experiment, bearing on pairs of spin-half particles in the singlet state (total angular momentum = 0):

$$|\psi\rangle = 2^{-1/2}(|+-\rangle - |-+\rangle)\,.$$

In the classical interpretation, quantum mechanics says that after the measurement of the component $\sigma_z^{(1)}$ of the spin 1 at the point A, the state vector is changed into $|\psi^*\rangle = |+-\rangle$ if the result of the experiment is $+\frac{1}{2}$ and $|\psi^*\rangle = |-+\rangle$ if it is $-\frac{1}{2}$. This implies that the result of the measurement of the component $\sigma_z^{(2)}$ of the spin 2 at an arbitrarily distant point B is completely determined as soon as $\sigma_z^{(1)}$ has been measured. The difficulty comes from the fact that quantum mechanics says nothing about how the spin 2 measured at B is influenced by the measurement of $\sigma_z^{(1)}$ at A: this influence can even be instantaneous. To preserve a principle of local causality, EPR suggested that the quantum-mechanical description of the physical reality is not complete. This led to think that the distant correlations between two measurements can be explained by "hidden variables" which propagate some information about the actual spin of the particles. Bell [3] later proved that any local hidden-variable theory and quantum mechanics predict completely different results for a given set of experiments involving such pairs of particles. Those differences allow to discriminate experimentally between quantum mechanics and local hidden-variable theories [4] at least if one takes the precaution to set rapidly enough, and independently, the instruments [5,6].

But any such experiment needs to obtain the results of measurements made simultaneously at two points A and B distant from each other. These results cannot be obtained together at the very instant of the measurement: one can only obtain immediately one of the two results (say at A) and after a time $t \geqslant |AB|/c$, the second one coming from B. The other possibility is to obtain the two results simultaneously at the middle M of AB, after a time $t' \geqslant |AB|/2c$. This noticing is the central argument of a recent paper [7], whose author shows that quantum mechanics does not predict that the state of particle 2 is determined instantaneously by the measurement on 1, but only that the conditional probability to find the result $\sigma_z^{(2)} = +\frac{1}{2}$ *if* $\sigma_z^{(1)} = -\frac{1}{2}$, and conversely, is equal to 1. Such a prediction can only be tested when it is possible to compare the two results, and not as soon as $\sigma_z^{(1)}$ has been measured. The present paper is concerned by drawing out, from the latter remarks, some general consequences on our conceptions of physical knowledge.

For this, we first define what we mean by a simple

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event, then build up the composed kind of event which can be reached in a Bohm-type experiment and then set down two assumptions about the events, from which generalized Bell's inequalities can be derived. Finally, the meaning of the renouncement to one of these two assumptions is discussed, and interpreted by making use of the many-worlds formulation of quantum mechanics.

To define an event, we consider Bell's simplified experimental arrangement [8]. It is made of three distant points A, B and M with three inputs and three outputs. The central input (at M) sets the experiment off at time t_1 and corresponds in Bohm's thought-experiment to the emission of the pair of particles. The central output can have the two values ±1 for good or wrong start of the experiment. At time $t_1 + T$, with $T \geqslant |AB|/2c$, the other outputs (each ±1) appear at A and B, corresponding to the results of the two spin measurements. Just before $t_1 + T$, at time $t_1 + T - \delta$, the inputs a and b are injected at A and B, δ being such that $c\delta \ll |AB|$. Such inputs correspond to the setting of the instruments (let us say the choice of the angles of rotation of the Stern–Gerlach magnets).

Definition 1. The simple event $\mathcal{E}_A$ is defined by the following set of propositions:

$$\mathcal{E}_A = \{(\mathrm{E}_A, r_A); (\mathrm{a}, t_\mathrm{a}); \mathrm{O}; (\mathrm{o}_A, t_{\mathrm{o}A}); L_A\} ,$$

where E_A states the space of the possible inputs at A, r_A the position of A, a the input, t_a the instant of the input, O the space of the possible outputs (±1), o_A the output at A, $t_{\mathrm{o}A}$ the instant of the output and $L_A = (r_{LA}, t_{LA})$ the space–time position of the recording of the previous propositions. We have of course $t_{LA} \geqslant t_{\mathrm{o}A} \geqslant t_\mathrm{a}$. $\mathcal{E}_M$, $\mathcal{E}_B$, and other events are defined in the same way. In the previous experimental set-up, $t_{\mathrm{o}M} = t_1$, $t_{\mathrm{o}A} = t_{\mathrm{o}B} = t_1 + T$, and $t_\mathrm{a} = t_\mathrm{b} = t_1 + T - \delta$.

Remark: The event is usually defined merely by stating the output and its space–time position. The present definition includes three space–time positions corresponding to the input, to the output, and to the recording. Each of these components does not define an event alone, but only when associated to the others.

Definition 2. The composed event $\mathcal{E}^*$ which can be written: $\mathcal{E}^* = \mathcal{E}_{A1} \oplus \mathcal{E}_{A2} \oplus ... \oplus \mathcal{E}_{An}$ is the set of all the sets of propositions defining the $\mathcal{E}_{Ai}$ recorded at a given single space–time position: $L = (r_L, t_L)$.

For instance, the event $\mathcal{E}_C^* = \mathcal{E}_A \oplus \mathcal{E}_B \oplus \mathcal{E}_M$ is recorded at $L_C = L_A = L_B = L_M = (r_{LC}, t_{LC})$. If we choose $r_{LC} = r_M$ (position at the middle of AB), we have: $t_{LC} \geqslant t_1 + |AB|/2c$ because the signals corresponding to the outputs take at least a time $|AB|/2c$ to go from A or B to M.

We now set down our two assumptions:

Assumption 1. Retrodiction from a composed event. If a composed event $\mathcal{E}^*$ has been recorded at $L = L_{Ai} (i = 1, ..., n)$, it is meaningful to associate to it the set of events $\{\mathcal{E}'_{Ai} (i = 1, ..., n)\}$ such that:

$$\mathcal{E}'_{Ai} = \{(\mathrm{E}_{Ai}, r_{Ai}); (\mathrm{a}_i, t_{\mathrm{a}i}); \mathrm{O}; (\mathrm{o}_{Ai}, t_{\mathrm{o}Ai}); L'_{Ai}\} ,$$

where $L'_{Ai} = (r_{Ai}, t_{\mathrm{o}Ai})$.

Remark: This assumption allows one to associate with the composed event $\mathcal{E}_C^*$ recorded at L_C the set of three events $\mathcal{E}'_A$, $\mathcal{E}'_B$, $\mathcal{E}'_M$ including the same elements as $\mathcal{E}_A$, $\mathcal{E}_B$, $\mathcal{E}_M$, and thus as $\mathcal{E}_C^*$, but for the positions of recording which are separated for the primed events: $L'_A = (r_A, t_1 + T)$, $L'_B = (r_B, t_1 + T)$, $L'_M = (r_M, t_1)$.

As was pointed out in the introduction, the only set of events which can be reached experimentally in the typical set-up is $\mathcal{E}_C^*$. Assumption 1 attributes a meaning to the set $\{\mathcal{E}'_A, \mathcal{E}'_B, \mathcal{E}'_M\}$ though its elements cannot be compared experimentally.

Assumption 2. Locality of the events. Consider the pair of events $\mathcal{E}_{Aj}$, $\mathcal{E}_{Aj'}$, such that $L_{Aj} \neq L_{Aj'}$. If:

$$c^2(t_{\mathrm{a}j} - t_{\mathrm{a}j'})^2 - |r_{Aj} - r_{Aj'}|^2 < 0 ,$$

$$c^2(t_{\mathrm{o}Aj} - t_{\mathrm{o}Aj'})^2 - |r_{Aj} - r_{Aj'}|^2 < 0 ,$$

and

$$c^2(t_{LAj} - t_{LAj'})^2 - |r_{LAj} - r_{LAj'}|^2 < 0 ,$$

the two events are independent of each other. Moreover, $\mathcal{E}_{Aj}$ is independent of $\mathcal{E}_{Aj'}$, even if some component of $\mathcal{E}_{Aj}$ is included in one of the *backward* light cones of $\mathcal{E}_{Aj'}$.

We say that an event $\mathcal{E}_{Aj}$ is independent of $\mathcal{E}_{Aj'}$ iff the components of $\mathcal{E}_{Aj}$ are not influenced by the components of $\mathcal{E}_{Aj'}$. In particular, o_{Aj} is neither influenced by $\mathrm{a}_{j'}$ nor by $\mathrm{o}_{Aj'}$ in such case.

Remarks: (1) It must be noticed that, according to this assumption, the origin of one of the light cones which define the locality of the events $\mathcal{E}_{Ai}$, is the space–time position of the recording, and not only the position of the output. Consider the situation in which

$$c^2(t_{\mathrm{a}j} - t_{\mathrm{a}j'})^2 - |r_{Aj} - r_{Aj'}|^2 < 0 ,$$

and

$$c^2(t_{oAj} - t_{oAj'})^2 - |r_{Aj} - r_{Aj'}|^2 < 0 \,,$$

while

$$c^2(t_{LAj} - t_{LAj'})^2 - |r_{LAj} - r_{LAj'}|^2 \geqslant 0 \,.$$

Assumption 2 does not rule out the possibility of an interaction between $\mathcal{E}_{Aj}$ and $\mathcal{E}_{Aj'}$.

(2) Consider the three events $\mathcal{E}'_A$, $\mathcal{E}'_B$, $\mathcal{E}'_C$. The events $\mathcal{E}'_A$ and $\mathcal{E}'_B$ are space-like separated, independent, and their components are thus independent too, though these components also belong to $\mathcal{E}^*_C$ (except for the position of the recording). The mere possibility to associate the events $\mathcal{E}'_A$, $\mathcal{E}'_B$, $\mathcal{E}'_M$ to $\mathcal{E}^*_C$ therefore implies that the components of the events $\mathcal{E}_A$, $\mathcal{E}_B$, elements of $\mathcal{E}^*_C$ become necessarily independent. Conversely, without the possibility to associate $\mathcal{E}'_A$, $\mathcal{E}'_B$, $\mathcal{E}'_M$ to $\mathcal{E}^*_C$, assumption 2 does not imply that the components of $\mathcal{E}_A$ and $\mathcal{E}_B$ are independent.

It is now very easy to demonstrate that assumptions 1 and 2 yield the Clauser–Holt–Horne–Shimony inequality [4,8]. Let us start from the composed event which can actually be recorded in the standard experimental set-up: $\mathcal{E}^*_C = \mathcal{E}_A \oplus \mathcal{E}_B \oplus \mathcal{E}_M$. Assumption 1 allows to associate the set $\{\mathcal{E}'_A, \mathcal{E}'_B, \mathcal{E}'_M\}$ to $\mathcal{E}^*_C$. The space–time positions of $\mathcal{E}'_A$ and $\mathcal{E}'_B$ are such that $\mathcal{E}'_A$ is outside the forward light cones of $\mathcal{E}'_B$ and conversely. The events $\mathcal{E}'_A$ and $\mathcal{E}'_B$ are thus, from assumption 2, independent of each other. In particular, o_A cannot depend either on b or on o_B. The converse is also true: o_B depends neither on a nor on o_A. Assumption 2 however allows $\mathcal{E}'_A$ and $\mathcal{E}'_B$ to be both dependent on a given set S of events including $\mathcal{E}'_M$ and being such that all its elements have forward light cones which include $\mathcal{E}'_A$ and $\mathcal{E}'_B$. S does not contain $\mathcal{E}'_A$ and $\mathcal{E}'_B$ themselves for $\mathcal{E}'_B$ is out of the forward light cones of $\mathcal{E}'_A$, and conversely. Therefore, the elements of S cannot depend on a or on b.

S being fixed, the joint additional probability $P(o_A, o_B|a,b,S)$ can then be separated into a product of independent factors:

$$P(o_A, o_B|a,b,S) = P(o_A|a,S)P(o_B|b,S) \,. \qquad (1)$$

If we now consider a probability distribution $p(S_n)$ over the possible sets of events S_n preceding $\mathcal{E}'_A$ and $\mathcal{E}'_B$, the averaged probability is:

$$\bar{P}(o_A, o_B|a,b) = \sum_n p(S_n)P(o_A, o_B|a,b,S_n) \,. \qquad (2)$$

As was demonstrated in ref. [8], the two equations (1) and (2) yield the Clauser–Holt–Horne–Shimony inequality which contradicts in many situations the corresponding quantum-mechanical predictions.

It is now very important to examine whether the former derivation can take place without using assumption 1. In such a case, it is not possible to associate $\mathcal{E}'_A$, $\mathcal{E}'_B$, $\mathcal{E}'_M$ to $\mathcal{E}^*_C$. The events $\mathcal{E}_A$ and $\mathcal{E}_B$ are not necessarily independent according to assumption 2, and thus o_A can be dependent on a, b, and o_B, and conversely. Moreover, the space–time position of recording of $\mathcal{E}_A$ and $\mathcal{E}_B$ is the common space–time position L_C. The set S defined previously can then include $\mathcal{E}_A$ and $\mathcal{E}_B$, and therefore its elements can depend on a and b. As a consequence, neither eq. (1) nor the CHHS inequality can be derived.

We have thus shown in a similar way as the one of d'Espagnat [9,10] that a set of general assumptions making no reference to any existing theory of microphysics implies a contradiction with quantum mechanics. The difference with ref. [9] however, is that in the present paper, the axioms bear on the very events and not on the systems whose interaction with the instruments is supposed to yield the events. This has allowed to reduce the number of assumptions by making pointless any hypothesis about properties of a system.

It is necessary at this stage to study how avoiding the contradiction within the former framework. This purpose can only be reached by ruling out one (or two) of the two assumptions that imply such a contradiction. The first possibility is well known: it consists in admitting the non-separability of the events $\mathcal{E}'_A$ and $\mathcal{E}'_B$, i.e. to rule out assumption 2. The second possibility is however much more unusual. We have indeed demonstrated that the CHHS inequalities can only be drawn from the consideration of the events $\mathcal{E}'_A$ and $\mathcal{E}'_B$ associated "a posteriori" to the composed event $\mathcal{E}^*_C$.

It thus suffices to rule out assumption 1 to avoid the contradiction with quantum mechanics. This is of course very hard to understand because we must accept the idea according to which from a given recorded event $\mathcal{E}^*_C$, it is impossible to give any meaning to the former events $\mathcal{E}'_A$ and $\mathcal{E}'_B$ whose space–time positions of recording are that of the outputs. Such conclusion can however be justified by the fact that, contrary to the case of the events $\mathcal{E}_A$ and $\mathcal{E}_B$ composing $\mathcal{E}^*_C$, there is definitely no experimental way of comparing $\mathcal{E}'_A$ and

$\mathcal{E}'_B$: the pair $\{\mathcal{E}'_A, \mathcal{E}'_B\}$ cannot thus be defined operationally. From this viewpoint, the EPR correlations are no more surprising: if one assumes that quantum-mechanical predictions do not bear on a pair of events $\{\mathcal{E}'_A, \mathcal{E}'_B\}$ which cannot be compared experimentally but only on $\{\mathcal{E}_A, \mathcal{E}_B\}$, an influence of $\mathcal{E}_A$ on $\mathcal{E}_B$ or conversely is locally acceptable since it does not need to travel faster than light.

We thus see that to give predictions which are not in contradiction with quantum mechanics, we must either renounce to a locality principle or to a retrodiction principle. At the present stage of science, the locality principle is an important element of general physical theories such as relativity, and is probably impossible to falsify with any "EPR device" [11]. On the contrary, the retrodiction principle is only a variant of an ontological presupposition which states approximately: "A pair of separated events can exist even if there is no experimental way of reaching them together" (notice that we say nothing in this statement about the presence or absence of a "consciousness" being aware of the events, but only about the mere possibility of reaching them experimentally).

The choice which consists in renouncing the possibility of retrodiction about the events can however give rise to an apparent conceptual difficulty. To bring out this difficulty, consider that three experimenters are initially located at the points A, B and M. The first one records the event $\mathcal{E}'_A$, the second one $\mathcal{E}'_B$, and the third one $\mathcal{E}'_M$. Then, they travel (as slowly as they like) towards a meeting point, and at a time $t_{\text{comp}} \geqslant t_1 + T + |AB|/2c$, they compare their results. Each experimenter can assure that the results o_A, o_B, and o_M have not changed during the trip. Any modification of the result between the instant of the output and the instant of the comparison thus seems to be ruled out, and the correlations between the measurements again require to renounce the locality principle.

An analysis of Bohm's experiment by using the many-worlds formulation of quantum mechanics can help us to rule out the former apparent contradiction between the concept of continuity of an observer and the giving up of the retrodiction principle. We shall use for that Everett's original notations [12]. Consider the system S^* described by a state vector belonging to the Hilbert space H_{S^*}:

$$|\psi S^*\rangle = 2^{-1/2}(|+-\rangle - |-+\rangle) = |\phi_{+-}\rangle + |\phi_{-+}\rangle . \quad (3)$$

Consider then the two observers M_A and M_B, the first being located at A and the second at B. These observers are respectively described by the state vectors: $|\psi^A[\vdots\vdots]\rangle$ and $|\psi^B[\vdots\vdots]\rangle$ belonging to H_A and H_B, where the two rows in the brackets stand for memory configurations. The upper row corresponds for a given observer to the past results of observations of a system S^*, and the lower row to the past results of observations of an other observer's memory state (the distinction is of course only conventional).

The composite system made of S^*, M_A and M_B is described by a state belonging to the tensor product $H_{S^*} \otimes H_A \otimes H_B$. Before the measurement of the system carried out by both observers, we can write the total state:

$$|\psi^{S^*+A+B}\rangle = |\psi^{S^*}\rangle|\psi^A[\vdots\vdots]\rangle|\psi^B[\vdots\vdots]\rangle . \quad (4)$$

Let us assume that the two measurements (of S^* by M_A and of S^* by M_B) are quasi-simultaneous, but that M_A operates first. Just after the measurement of S^* by M_A, the total state becomes:

$$|\psi^{S^*+A'+B}\rangle = (|\phi_{+-}\rangle|\psi^A[\vdots\vdots\,{}^{+1}]\rangle + |\phi_{-+}\rangle|\psi^A[\vdots\vdots\,{}^{-1}]\rangle)|\psi^B[\vdots\vdots]\rangle . \quad (5)$$

It is clear that, though the measurement of S^* has been performed by M_A, the probabilities for the observer M_B to obtain +1 or −1 are still equal. This absence of influence of the measurement made by M_A on the result obtained by M_B is due to the absence of any discontinuous transitions from the state $|\psi^{S^*}\rangle$ to one of the states $|\phi_{+-}\rangle$ or $|\phi_{-+}\rangle$. As pointed out by Everett, there is no "transition from possible to actual" in his formulation. Every possible result indeed is actual.

After the measurement of S^* by M_B, we obtain:

$$|\psi^{S^*+A'+B'}\rangle = |\phi_{+-}\rangle|\psi^A[\vdots\vdots\,{}^{+1}]\rangle|\psi^B[\vdots\vdots\,{}^{-1}]\rangle + |\phi_{-+}\rangle|\psi^A[\vdots\vdots\,{}^{-1}]\rangle|\psi^B[\vdots\vdots\,{}^{+1}]\rangle . \quad (6)$$

At this stage, the states of the observers appear (but only to us, because we have the position of "super-observers") as correlated. In fact, this correlation will have empirical consequences only at the instant when M_A is able to know what result M_B has obtained and conversely. Consider indeed that the observer M_B has reached M_A. After the measurement of the observers by each other, the state (6) becomes:

$$|\psi^{S^*+A''+B''}\rangle = |\phi_{+-}\rangle|\psi^A[::{}^{+1}_{-1}]\rangle|\psi^B[::{}^{-1}_{+1}]\rangle$$
$$+\ |\phi_{-+}\rangle|\psi^A[::{}^{-1}_{+1}]\rangle|\psi^B[::{}^{+1}_{-1}]\rangle\ . \qquad (7)$$

To summarize the former results, we notice that the fact that, in a given world, M_A has observed the result o_A does not influence the measurement performed by M_B. It can only be said that if M_A has obtained o_A, he is in a world where it is impossible for him to meet the observer M_B who has not measured $-o_A$. Conversely, the observer M_B who has observed $+o_A$ will never be able to meet that particular observer M_A who has observed o_A, but only the one (living in the other-world) who has observed $-o_A$.

We now understand how the giving up of the retrodiction principle can be compatible with the continuity of the observers. When M_A has compared his own result to the result of M_B, M_A can be sure that, in the world where he lives, he had already obtained the same result o_A at the instant of the measurement of the system S^*. This ensures the continuity of the observer M_A *in a given world.* But M_A cannot say that the observation of o_A is the only result which has been obtained at A. The two results o_A and $-o_A$ have indeed been obtained at the instant of the measurement, giving rise to a splitting of the world into two branches. The latter circumstance implies that the event of type $\mathcal{E}'_A$ to associate to $\mathcal{E}^*_C$ is undefined, precluding a statement of retrodiction such as assumption 1.

It therefore becomes clear that, as already pointed out by Page [7], the EPR paradox can be solved without giving up the locality principle, if the quantum-mechanical description of the correlations takes into account a complete definition of the experimental conditions. An analysis of the EPR correlations in terms of events has moreover challenged a very general assumption of human knowledge: the possibility of retrodiction about an event. Though very puzzling, a giving up of such an assumption has been shown to lead to no contradiction.

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